

## PAPER-1 (B.E. / B.TECH)

# QUESTIONS & SOLUTIONS

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📅 18 March, 2021

🕒 09:00 am to 12 Noon

SHIFT-1



Duration : 3 Hours

Max. Marks : 300

## SUBJECT - MATHEMATICS

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**ROUNIK ROUSHAN**

4 Years Classroom Program (till 10<sup>th</sup>)  
Completed IIT Syllabus in his  
Class 10<sup>th</sup> at Vidyapeeth Academy.



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**MATHEMATICS**

1. Let circle

$$C_1 : x^2 + y^2 = 1$$

$$C_2 : x^2 + y^2 - 2y = 1$$

$$C_3 : x^2 + y^2 - 2x = 1$$

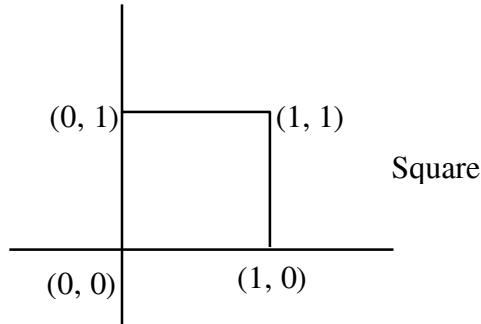
$$C_4 : x^2 + y^2 - 2x - 2y + 1 = 0$$

figure formed by joining centres of  $C_1, C_2, C_3, C_4$  is

- (1) square                      (2) rectangle                      (3) rhombus                      (4) parallelogram

Ans. (1)

Sol. Centres  $(0, 0), (1, 0), (0, 1), (1, 1)$



2. The values of a and b for which the function  $f(x) = \begin{cases} \frac{1}{|x|} & , |x| \geq 1 \\ ax^2 + b & , |x| < 1 \end{cases}$  is differentiable, are respectively

- (1)  $\frac{1}{2}, \frac{3}{2}$                       (2)  $-\frac{1}{2}, \frac{3}{2}$                       (3)  $\frac{1}{2}, -\frac{3}{2}$                       (4)  $1, -2$

Ans. (2)

Sol.  $f(x)$  is continuous at  $x = 1 \Rightarrow 1 = a + b$   
 $f(x)$  is differentiable at  $x = 1 \Rightarrow -1 = 2a$   
 $\Rightarrow a = -\frac{1}{2} \therefore b = \frac{3}{2}$

3. Find the sum of all the four digit numbers formed by using the digits 1,2,2,3.

- (1) 36664                      (2) 26664                      (3) 24                      (4) 46632

Ans. (2)

Sol. ....1 =  $\frac{3!}{2!}$   
 .....2 =  $3!$   
 .....3 =  $\frac{3!}{2!}$   
 sum of digits at unit's place =  $3 + 12 + 9 = 24$   
 sum of all four digits numbers =  $24(1111) = 26664$

4. Find the number of points of intersection of the curves :

$$S_1 : x^2 + y^2 - 22x + 10y + 137 = 0 \text{ \&}$$

$$S_2 : x^2 + y^2 - 10x - 10y + 41 = 0$$

- (1) 1                                      (2) 2                                      (3) 0                                      (4) 3

Ans. (1)

Sol.  $S_1 : (x - 11)^2 + (y - 5)^2 = 9 \rightarrow C_1 = (11, 5)$

$$S_2 : (x - 5)^2 + (y - 5)^2 = 9 \rightarrow C_2 = (5, 5)$$

$$r_1 = 3 \text{ \& } r_2 = 3$$

$$d(C_1 C_2) = \sqrt{(11 - 5)^2} = 6$$

$$r_1 + r_2 = 6$$

\(\therefore\) Circles touch externally

Hence, 1 point of intersection.

5. If  $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \dots + a_{40}x^{40}$  then the value of  $a_1 + a_2 + \dots + a_{37}$  is :

- (1)  $2^{18}(2^{19} - 19)$                       (2)  $2^{19}(2^{20} - 21)$                       (3)  $2^{18}(2^{20} - 38)$                       (4)  $2^{20}(2^{20} - 21)$

Ans. (2)

Sol. Put  $x = 1, -1$  and subtract

$$4^{20} - 2^{20} = (a_0 + a_1 + \dots + a_{40}) - (a_0 - a_1 + \dots)$$

$$\Rightarrow 4^{20} - 2^{20} = 2(a_1 + a_3 + \dots + a_{39})$$

$$\Rightarrow a_1 + a_3 + \dots + a_{37} = 2^{39} - 2^{19} - a_{39}$$

$$a_{39} = \text{coeff of } x^{39} \text{ in } (1 + x + 2x^2)^{20} = {}^{20}C_1 2^{19}$$

$$\begin{aligned} \Rightarrow a_1 + a_3 + \dots + a_{37} &= 2^{39} - 2^{19} - 20(2^{19}) \\ &= 2^{39} - 21(2^{19}) = 2^{19}(2^{20} - 21) \end{aligned}$$

6. Find the sum of the series :

$$S = \frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \dots + \frac{1}{201^2 - 1}$$

- (1)  $\frac{1}{4}$                                       (2)  $\frac{25}{101}$                                       (3)  $\frac{26}{99}$                                       (4)  $\frac{26}{102}$

Ans. (2)

Sol.  $S = \sum_{r=1}^{100} \frac{1}{(2r+1)^2 - 1} = \sum_{r=1}^{100} \frac{1}{(2r+1) \cdot 2(r)}$

$$\therefore S = \frac{1}{4} \sum_{r=1}^{100} \left[ \frac{1}{r} - \frac{1}{r+1} \right]$$

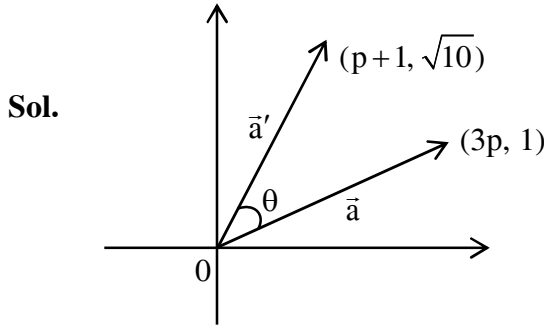
$$S = \frac{1}{4} \left( \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{100} - \frac{1}{101}\right) \right)$$

$$\therefore S = \frac{1}{4} \left[ \frac{100}{101} \right] = \frac{25}{101}$$

7. A vector  $\vec{a}$  has components  $3p$  and  $1$  with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system,  $\vec{a}$  has components  $(p + 1)$  and  $\sqrt{10}$ , then find the value of 'p'

- (1)  $-\frac{5}{4}$                       (2)  $\frac{4}{5}$                       (3)  $1$                       (4)  $-1$

Ans. (4)



given  $\vec{a} = 3p\hat{i} + \hat{j}$ ,  $\vec{a} = (p + 1)\hat{i} + \sqrt{10}\hat{j}$   
 $|\vec{a}| = |\vec{a}'|$ , (No Change in magnitude )

$$\Rightarrow \sqrt{9p^2 + 1} = \sqrt{(p + 1)^2 + 10}$$

$$9p^2 + 1 = p^2 + 2p + 1 + 10$$

$$8p^2 - 2p - 10 = 0$$

$$4p^2 - p - 5 = 0$$

$$(4p - 5)(p + 1) = 0$$

$$p = -1, p = \frac{5}{4}$$

8. The value of  $\int \frac{(2x - 1) \cos \sqrt{(2x - 1)^2 + 4}}{\sqrt{4x^2 - 4x + 5}} dx$  is

- (1)  $\frac{1}{2} \sin(\sqrt{4x^2 - 4x + 5}) + C$                       (2)  $\sin(\sqrt{4x^2 - 4x + 5}) + C$   
 (3)  $\tan(\sqrt{4x^2 - 4x + 5}) + C$                       (4) None of these

Ans. (1)

Sol. Put  $\tan^{-1}\left(\frac{2x - 1}{2}\right) = \theta$

$$\Rightarrow 2x - 1 = 2 \tan \theta$$

$$\Rightarrow I = \int \frac{2 \tan \theta \cos(2 \sec \theta)}{2 \sec \theta} \cdot \sec^2 \theta d\theta$$

$$\Rightarrow I = \int \tan \theta \sec \theta \cos(2 \sec \theta) d\theta$$

Put  $2 \sec \theta = t \Rightarrow 2 \sec \theta \tan \theta d\theta = dt$

$$\Rightarrow I = \frac{1}{2} \int \cos t dt = \frac{\sin t}{2} + c = \frac{\sin(2 \sec \theta)}{2} + C$$

$$= \frac{1}{2} \sin(\sqrt{4x^2 - 4x + 5}) + C$$

9. Let  $a|z|^2 + \bar{\alpha}z + \bar{z}\alpha + d = 0$  represents the equation of the circle when

(1)  $\alpha^2 - ad > 0$  such that  $a \in \mathbb{R}$

(2)  $\alpha^2 - ad \geq 0$  such that  $a \in \mathbb{R} - \{0\}$

(3)  $\alpha^2 - ad = 0$  for  $\forall a \in \mathbb{R}$

(4)  $\alpha^2 - ad \leq 0$  for  $\forall a \in \mathbb{R}$

Ans. (2)

Sol.  $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0$  ..... (i)

$$z\bar{z} + \frac{\alpha}{a}\bar{z} + \frac{\bar{\alpha}}{a}z + \frac{d}{a} = 0 \Rightarrow \text{circle}$$

centre =  $-\frac{\alpha}{a}$ ,  $r = \sqrt{|\alpha|^2 - c}$

$$\Rightarrow \left| \frac{\alpha}{a} \right|^2 - \frac{d}{a} \geq 0 \text{ for equation (i) to represents a circle}$$

$$\Rightarrow |\alpha|^2 - ad \geq 0$$

10. Let the system of equation  $\alpha u + \beta v + \gamma w = 0$ ,  $\beta u + \gamma v + \alpha w = 0$ ,  $\gamma u + \alpha v + \beta w = 0$  has non trivial

solution and  $\alpha, \beta, \gamma$  are distinct roots of  $x^3 + ax^2 + bx + c = 0$  find value of  $\frac{a^2}{b}$

(1) 0

(2) 1

(3) 2

(4) 3

Ans. (1)

Sol. 
$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$$\Rightarrow \alpha^2 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

$$\therefore \alpha + \beta + \gamma = 0 \text{ or } \alpha = \beta = \gamma$$

Here  $\alpha + \beta + \gamma = 0$

$$\therefore (\alpha, \beta, \gamma \text{ distinct})$$

$$\therefore a = 0$$

11. Let  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt{1-x}$  and function  
 $f(x) + g(x), f(x) - g(x), \frac{f(x)}{g(x)}, \frac{g(x)}{f(x)}, g(x) - f(x)$

Find the set of value of 'x' which is common to the domain of all the five functions.

- (1)  $x \in (0,1)$                       (2)  $x \in [0,1]$                       (3)  $x \in [0,1)$                       (4)  $x \in (0,1]$

Ans. (1)

Sol.  $D_f = [0, \infty)$

$$D_g = (-\infty, 1]$$

$$D_{f+g} = D_{f-g} = D_{\frac{f}{g}} = D_{\frac{g}{f}} = [0, \infty) \cap (-\infty, 1] = [0, 1]$$

for  $\frac{f}{g}$  and  $\frac{g}{f}$  to be defined,  $x \neq 0, 1 \Rightarrow$  common domain =  $(0, 1)$

12. If  $(100)^\alpha - (199)^\beta = (100)(100) + (99)(101) + (98)(102) + \dots + (1)(199)$ . Then find the slope of line joining  $(\alpha, \beta)$  and origin.

- (1) 540                      (2) 550                      (3) 530                      (4) 545

Ans. (2)

Sol.  $RHS = \sum_{r=0}^{99} (100-r)(100+r)$

$$= (100)^3 - \frac{99 \times 100 \times 199}{6} = (100)^3 - (1650) 199$$

$$LHS = (100)^\alpha - (199)^\beta$$

$$\text{So, } \alpha = 3, \beta = 1650$$

$$\text{Slope} = \tan \theta = \frac{\beta}{\alpha}$$

$$\tan \theta = 550$$

13. Let the straight lines are  $y = (mx + 1)$ ,  $3x + 4y = 9$ . Find number of integral values of m for which abscissa of point of intersection is integer

- (1) 1                      (2) 2                      (3) 3                      (4) 0

Ans. (2)

Sol.  $3x + 4(mx + 1) = 9$

$$x(3 + 4m) = 5$$

$$x = \frac{5}{(3 + 4m)}$$

$$(3 + 4m) = \pm 1, \pm 5$$

$$4m = -3 \pm 1, -3 \pm 5$$

$$4m = -4, -2, -8, 2$$

$$m = -1, -\frac{1}{2}, -2, \frac{1}{2}$$

Two integral value of m



**Sol.**  $t_r(2A + B) = 1 \Rightarrow 2t_r(A) + t_r(B) = 1$  .....(1)

$t_r(A - 2B) = 3 \Rightarrow t_r(A) - 2t_r(B) = 3$  .....(2)

$\Rightarrow t_r(A) - 2[1 - 2t_r(A)] = 3$

$\Rightarrow t_r(A) = 1, t_r(B) = -1$

$\therefore t_r(A) - t_r(B) = 2$

**17.** Let  $y = (3\sqrt{2})x - 1$  be given line then find equation of straight line passes through A(1, 3) and makes an angle  $\tan^{-1}\sqrt{2}$  with given line

(1)  $5y + 4\sqrt{2}x = 4\sqrt{2} + 15$

(2)  $5y - 4\sqrt{2}x = 4\sqrt{2} + 15$

(3)  $4\sqrt{2}x - 5y = 4\sqrt{2} - 15$

(4) None of these

**Ans.** (1)

**Sol.**  $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\sqrt{2} = \left| \frac{m_1 - 3\sqrt{2}}{1 + 3\sqrt{2}m_1} \right|$

$\Rightarrow \pm\sqrt{2}(1 + 3\sqrt{2}m_1) = (m_1 - 3\sqrt{2})$

$\sqrt{2} + 6m_1 = m_1 - 3\sqrt{2}$  [for positive sign]

$m_1 = \frac{-4\sqrt{2}}{5}$

$-\sqrt{2} - 6m_1 = m_1 - 3\sqrt{2}$

$2\sqrt{2} = 7m_1 \Rightarrow m_1 = \frac{2\sqrt{2}}{7}$

**18.** Find number of times, the digit '3' appears while writing integers from 1 to 1000.

**Ans.** 300

**Sol.**  $\underline{3} \_ \_ = (9 \times 9) \times 1$

$\_ \underline{3} \_ = (9 \times 9) \times 1$

$\_ \_ \underline{3} = (9 \times 9) \times 1$

$\underline{3} \underline{3} \_ = (9) \times 2$

$\underline{3} \_ \underline{3} = (9) \times 2$

$\_ \underline{3} \underline{3} = (9) \times 2$

$\underline{3} \underline{3} \underline{3} = (1) \times 3$

total way =  $243 + 54 + 3 = 300$



19. Let  $P_1$  be the plane  $x - 2y + 2z - 3 = 0$  and  $A(1, 2, 3)$  lie on it. Let there be another plane  $P_2$  which is parallel to  $P_1$  and at unit distance from  $A$ . If  $P_2$  is  $ax + by + cz + d = 0$  the find positive value of  $\left(\frac{b-d}{c-a}\right)$

Ans. 4

Sol. Let  $P_2 : x - 2y + 2z + \lambda = 0$

$$\left| \frac{1-4+6+\lambda}{\sqrt{1+4+4}} \right| = 1$$

$$|\lambda + 3| = 3$$

$$(\lambda + 3) = \pm 3$$

$$\lambda = -3 \pm 3$$

$$\lambda = 0 \text{ or } -6$$

$$P_2 : x - 2y + 2z + 0 = 0$$

$$x - 2y + 2z - 6 = 0$$

$$\frac{b-d}{c-a} = \frac{(-2)-(0 \text{ or } -6)}{(2-1)} = -2 \text{ or } 4$$

so positive is 4

20. If  $f(x^2) + g(4-x) = 4x^3$  and  $g(x) + g(4-x) = 0$ , then the value of  $\int_{-4}^4 f(x^2) dx$

Ans. 512

Sol.  $I = 2 \int_0^4 f(x^2) dx \dots\dots\dots(i)$

$$\Rightarrow I = 2 \int_0^4 f((4-x)^2) dx \dots\dots\dots(ii)$$

Adding equation (i) & (ii)

$$2I = 2 \int_0^4 [f(x^2) + f(4-x)^2] dx \dots\dots\dots(iii)$$

Now using  $f(x^2) + g(4-x) = 4x^3 \dots\dots\dots(iv)$

$$x \rightarrow 4-x$$

$$f((4-x)^2) + g(x) = 4(4-x)^3 \dots\dots\dots(v)$$

Adding equation (iv) & (v)

$$f(x^2) + f(4-x^2) + g(x) + g(4-x) = 4(x^3 + (4-x)^3)$$

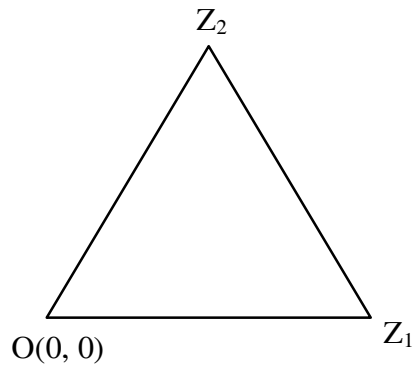
$$\Rightarrow f(x^2) + f(4-x^2) = 4[x^3 + (4-x)^3]$$

$$\text{Now, } I = 4 \int_0^4 (x^3 + (4-x)^3) dx = 512$$

21. If roots of  $x^2 + ax + 12 = 0$  and origin form equilateral triangle in complex plane, find  $|a|$  equal to

Ans. 6

Sol.



for equilateral triangle

$$Z_1^2 + Z_2^2 + O^2 = Z_1Z_2 + 0 + 0$$

$$(Z_1 + Z_2)^2 = 3Z_1Z_2$$

$$\Rightarrow (-a^2) = 3(12)$$

$$\Rightarrow a^2 = 36$$

$$a = -6 \text{ or } 6$$

$$|a| = 6$$

22. Average age of 25 teachers of a school is 40. If a teacher of age 60 year leave the school and a new teacher of 'x' age joined and new mean of their ages is 39 then find the value of 'x'

Ans. 35

Sol. 
$$\bar{x} = \frac{\sum_{i=1}^{25} x_i}{25} = 40$$

$$\sum_{i=1}^{25} x_i = 1000$$

when a teacher left school

$$\sum_{i=1}^{24} x_i = 1000 - 60 = 940$$

$$\bar{x}' = \frac{\sum_{i=1}^{24} (x_i) + x}{25}$$

$$39 \times 25 = 940 + x$$

$$x = 35$$

23. If  $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3} = \ell$  then

find  $6\ell + 1$ .

Ans. 4

Sol. 
$$\ell = \lim_{x \rightarrow 0} \frac{\left(x + \frac{x^3}{6} + \dots\right) - \left(x - \frac{x^3}{3} + \dots\right)}{x^3}$$

$$\therefore \ell = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

$$\therefore 6\ell = 4$$

24. Find number of solutions of equation

$$|\cot x| = \cot x + \frac{1}{\sin x} \text{ in } x \in [0, 2\pi]$$

Ans. 1

Sol. Case I :  $x \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$

$$\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \text{not possible}$$

Case : II  $x \in \left[\frac{\pi}{2}, \pi\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

$$-\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \frac{-2 \cos x}{\sin x} = \frac{1}{\sin x}$$

$$\Rightarrow \cos x = \frac{-1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$= 1$$

25. If  $\int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = f(x) + c$  and if  $f(0) = 0$  &  $f(1) = \frac{1}{k}$ , then find k.

Ans. 4

Sol. 
$$\int \frac{5x^8 + 7x^6}{(2x^7 + x^2 + 1)^2} dx = \int \frac{5x^8 + 7x^6}{x^{14} \left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)^2} dx$$

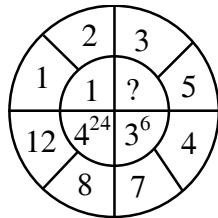
$$= \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(2 + \frac{1}{x^5} + \frac{1}{x^7}\right)} dx \quad : \text{put } 2 + \frac{1}{x^5} + \frac{1}{x^7} = t$$

$$= \int \frac{-dt}{t^2} = \frac{1}{t} + c \Rightarrow -\left(\frac{5}{x^6} + \frac{7}{x^8}\right) dx = dt$$

$$= f(x) = \frac{1}{2 + \frac{1}{x^5} + \frac{1}{x^7}} = \frac{x^7}{2x^7 + 1 + x^2}$$

$$f(x) = \frac{1}{4} = \frac{1}{k} \Rightarrow k = 4$$

26. Find the missing term in



Ans. 16

Sol.  $4^{24}$  has base 4 (= 12 - 8)

$3^6$  has base 3 (= 7 - 4)

(?) will have base 2 (= 5 - 3)

Power 24 =  $6 \times 4$  = (no. of divisor of 12)  $\times$  (no. of divisor of 8)

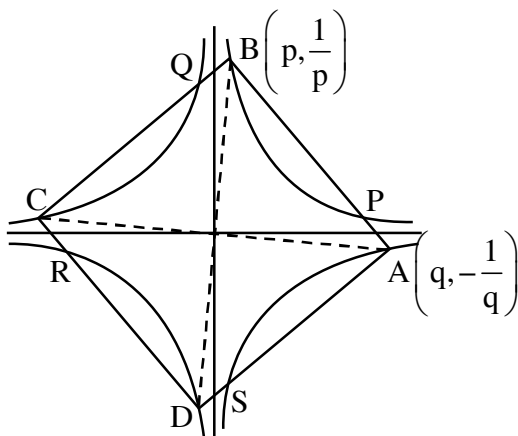
Power 6 =  $2 \times 3$  = (no. of divisor of 7)  $\times$  (no. of divisor of 4)

(?) will have power = (no. of divisor of 3)  $\times$  (no. of divisor of 5) =  $2 \times 2 = 4$

27. Four points lying on the curve  $x^2y^2 = 1$  form a square such that mid-points of sides also lie on the given curve. Find the square of area of the square

Ans. 80

Sol.



$$OA \perp OB$$

$$\Rightarrow \left(\frac{1}{p^2}\right)\left(-\frac{1}{q^2}\right) = -1$$

$$\Rightarrow p^2q^2 = 1$$

$$P \left( \frac{p+q}{2}, \frac{\frac{1}{p} - \frac{1}{q}}{2} \right) \text{ lies}$$

$$\text{on } x^2y^2 = 1$$

$$\Rightarrow (p+q)^2 \left( \frac{1}{p} - \frac{1}{q} \right)^2 = 16$$

$$\Rightarrow (p+q)^2(p-q)^2 = 16$$

$$\Rightarrow (p^2 - q^2)^2 = 16$$

$$\Rightarrow p^2 - \frac{1}{p^2} = \pm 4$$

$$\Rightarrow p^4 \pm 4p^2 - 1 = 0$$

$$\Rightarrow p^2 = \frac{\pm 4 \pm \sqrt{20}}{2} = \pm 2 \pm \sqrt{5}$$

$$\Rightarrow p^2 = 2 + \sqrt{5} \text{ or } -2 + \sqrt{5}$$

$$OB^2 = p^2 + \frac{1}{p^2} = 2 + \sqrt{5} + \frac{1}{2 + \sqrt{5}} \text{ or } -2 + \sqrt{5} + \frac{1}{-2 + \sqrt{5}} = 2\sqrt{5}$$

$$\text{Area} = 4 \left( \frac{1}{2} \right) (OA)(OB) = 2(OB)^2 = 4\sqrt{5}$$